# Statistical insights arising from point load testing of Danian limestone

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## ABSTRACT

The characterisation of the mechanical properties of the rocks can be carried out with conventional strength testing (e.g. unconfined compressive strength tests, tensile strength tests and triaxial tests) or with 'rapid' tests such as the point load test (PLT). Compared to the conventional tests the rapid tests are economical, quick and can also be carried out in situ; on the other hand, the results tend to be more scattered, correlations with the strength parameters depend on the rock type and the experience in using them in the design stage can be limited. This paper presents the results of PLTs carried out on Danian limestone and Maastrichtian chalk specimens, either as irregular lumps or as cylinders/disks, from multiple sites on Zealand in Denmark. The Danian limestone formations are weak sedimentary rocks with highly variable properties in terms of their strength, stiffness and in mass permeability. Their mechanical properties are governed by genesis, induration and the large variability of the fissuring and distribution of fissuring. By statistically analysing the results of the PLTs, it is possible to appreciate the impact that the height to equivalent diameter ratio has on the coefficient of variation for each induration class, which can therefore be used as guidance for specimen selection and/or preparation on site or in the laboratory.

## **KEYWORDS**

Point load test; Danian limestone; weak rock characterization; height to equivalent diameter ratio

## INTRODUCTION

The point load test (PLT) is an index test allowing for strength classification of rocks through measurement of the point load strength index  $I_{s(50)}$ , which can be correlated with the unconfined compressive strength, tensile strength and other properties of the rock. The PLT is a simple, fast and cost-effective way of obtaining information on the rock strength compared with other strength testing methods. In addition, PLTs may be performed both in situ and in the laboratory on different shapes and sizes of rock specimen and therefore may be preferable method of checking strength properties of rock. However, it is the understanding of the authors that these are seldom used for testing of Danian limestone. The results from PLTs are more variable compared with the results of the unconfined compressive strength tests, or triaxial tests on the rock: this is due to the variation of the size of tested specimen and the method of standardisation of the test result for the specimen size (Thuro & Plinninger, 2001).

The analysis presented below is based on a series of PLTs performed mainly on irregular rock lumps taken from the Danian limestone and Maastrichtian chalk cores tested in the laboratory and focuses on impact of the specimen size on the PLT results. The analysed sample of the PLT data excludes tests done on specimens of mixed indurations and with silicified parts or flint. Outliers defined as the  $I_{s(50)}$  values higher than mean  $I_{s(50)}$  value plus three standard deviations or lower than mean  $I_{s(50)}$  value minus three standard deviations are also eliminated from the sample. In the following, the terms 'data set' and 'sample' are used interchangeably. Approximately 480 results

of point load tests on limestone and chalk from two different sites on Zealand are presented below; only 20% of these were performed as axial tests while the others were performed as irregular lump tests.

Danian limestone and Maastrichtian chalk are weak sedimentary rocks; in Denmark their hardness is described by the degree of induration by geologists (Larsen et al., 1995). The degree of induration provides a very general indication of the rock strength.

## 1. ROCK CHARACTERIZATION BASED ON POINT LOAD TESTING

## **1.1. Danian limestone and Maastrichtian chalk**

The Danian limestones are subdivided into Copenhagen and Bryozoan limestone. Copenhagen limestone overlies Bryozoan limestone, which is underlain by Maastrichtian chalk. These weak rocks were formed from calcareous materials deposited in a marine environment and are horizontally bedded. While the Danian limestone is characterised by variable induration, the Maastrichtian chalk consists mainly of white chalk, which is only slightly indurated.

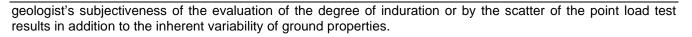
Local geological practice describes both Danian limestones and Maastrichtian chalk by the degree of induration, providing indirect information on the strength and compressibility of the rock material. The degree of induration is a semi-quantitative parameter subdivided in a five degree-scale as shown in Table 1 together with the corresponding ISRM rock grade as well as the indicative strength defined as compressive strength  $\sigma_c$  and point load strength index  $I_{s(50)}$ . The strength of the rock increases with increasing degree of induration as well as rock density.

For the purpose of this analysis, tests on specimens of Danian limestone and Maastrichtian chalk were grouped on the basis of the degree of induration.

Induration	Term	Description	ISRM Rock Grade	σ <sub>c</sub> , (MPa)	I <sub>s(50)</sub> , (MPa)
H1	Unlithified	The material can be easily formed by hand. Grainy material will fall apart when dry.	R0	0.25-1	-
H2	Slightly indurated	The material can easily by cut with a knife and can be scratched with a fingernail. Individual grains can be picked out with the fingers when the material is grainy.	R1	1-5	0.1-1
H3	Indurated	The material can be cut with a knife but cannot be scratched with a fingernail. Individual grains can be picked out with a knife when the material is grainy.	R2	5-25	1-4
H4	Strongly indurated	The material can be scratched with a knife. Individual grains do not come out with a knife. Fractures will follow grain surfaces.	R3, R4	25-100	2-5
H5	Very strongly indurated	The material cannot be scratched with a knife. Cracks and fracture surfaces will go through individual grains in grainy materials.	R5, R6	100-500	4-10

Table 1. Degree of induration according to Danish practice (Larsen et al., 1995) with corresponding indicative compressive strength  $\sigma_c$  and point load index  $I_{s(50)}$  based on Hansen & Foged (2002).

Figure 1 to Figure 3 present the variation of the moisture content and bulk density of the limestone and chalk with the degree of induration and the strength expressed as  $I_{s(50)}$ . High moisture content and low bulk density characterise the weaker H2 material, while the strongly indurated materials have lower moisture content and higher bulk density. As it may be observed from Figure 1 to Figure 3, the data overlap as limestone material with the same moisture content or bulk density may have different induration and strength. Only the data presented in Figure 1 from H2 indurated material mostly follows the indicative range of the point load index  $I_{s(50)}$  provided by Hansen & Foged (2002). As it can be appreciated from Figure 2 and Figure 3 within the tested H3 and H4 indurated material there are results with the lower  $I_{s(50)}$  than the lower range provided in Table 1. This maybe due to the



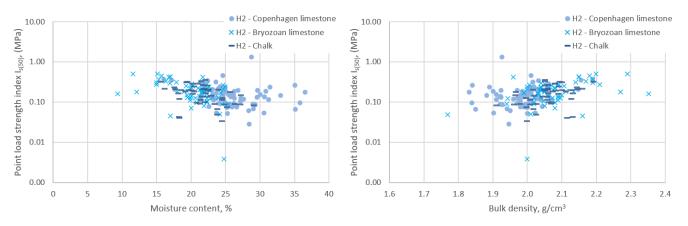


Figure 1. Normalised Point Load Strength Index  $I_{s(50)}$  vs moisture content and bulk density for H2 inducated material.

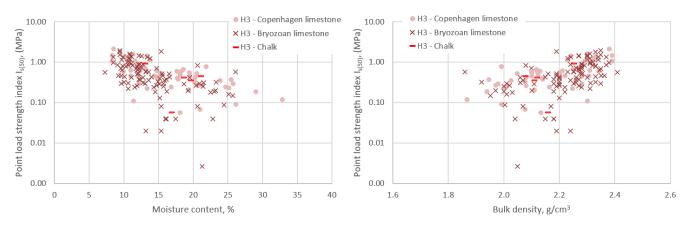


Figure 2. Normalised Point Load Strength Index  $I_{s(50)}$  vs moisture content and bulk density for H3 indurated material.

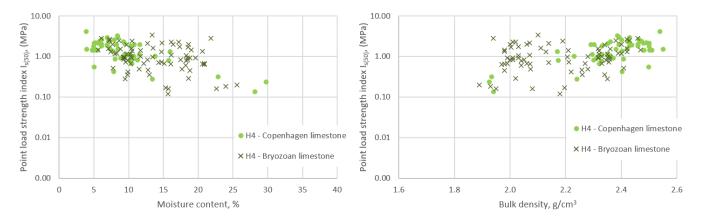


Figure 3. Normalised Point Load Strength Index  $I_{s(50)}$  vs moisture content and bulk density for H4 indurated material.

## 1.2. Point load testing

The PLT provides an indirect measure of the uniaxial strength of rock and can be performed on site, as well as in the laboratory on specimens of different shapes. While the test was originally introduced by Protodyaknov (1960), the first comprehensive coverage of PLT is normally attributed to Broch and Franklin (1972), whose contribution was the basis for the first ISRM suggested methods on this test. A description of the test procedure and the definition of the specimen dimensions are provided by the ISRM (1985). The data presented in this analysis was obtained in the laboratory mainly from irregular lump tests; for 20% of specimen axial tests were carried out.

The ISRM (1985) provides suggestions for specimen dimensions for different types of tests (see Figure 4). For axial and irregular lump tests the required ratio between the distance between the conical platen contact points D, referred to in the following as 'specimen height', and the lowest specimen height to width ratio is between 0.3 and 1.0, and preferably close to 1.0. The equivalent core diameter  $D_e$  has been calculated in accordance to ISRM (1985):

$$D_e = \sqrt{\frac{4 \times W \times D}{\pi}}$$
(1)

where all the symbols are defined in Figure 4.

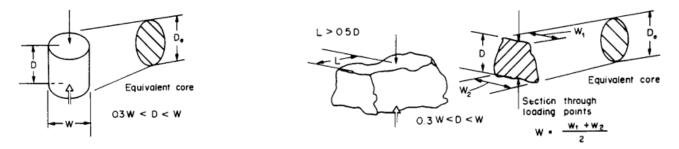


Figure 4. Specimen shape requirements for the axial test (left) and the irregular lump test (left) (ISRM, 1985).

The point load strength index is defined as the value of the point load strength that would have been measured by a diametral test with D=50mm (ISRM, 1985):

$$I_{s(50)} = \frac{P}{D_e^2} \times \left(\frac{D_e}{50}\right)^{0.45}$$
 (2)

where P is the failure load required to break the specimen, while  $D_e$  is defined in Figure 4. The first part of the equation is defined as uncorrected point load strength  $I_s$ , while the second part is the size correction factor. The  $I_{s(50)}$  normalises point load test results with respect to the specimen shape and size.

## 1.3. Data set summary, limitations and aim of this study

The aim of this analysis is to assess the impact of the specimen size defined as ratio between D and the equivalent core diameter  $D_e$  on variability of the  $I_{s(50)}$  results in order to achieve more precise and accurate PLT results for limestone and chalk materials depending on their degree of induration. It therefore focuses on the possibility of decreasing the epistemic uncertainty in the  $I_{s(50)}$  evaluation by choosing a specific specimen size for the test.

Specimens with silicified parts or flint parts were excluded to remove lithological heterogeneity of the limestone material; the outliers were also excluded from the analysis presented in section 2.2. Six tests were carried out on H5 material and the data were not analysed due to very small sample size. In addition, the  $I_{s(50)}$  values obtained from sample size with less than ten tests were eliminated, as the ISRM (1985) suggests to carry at least ten tests per sample; for this analysis a sample is defined by degree of induration and D/D<sub>e</sub> ratio. It is noted that too few

tests were available for the  $D/D_e$  ratio >0.8 for them to be statistical meaningful, which is a limitation of the data set assessed herein.

Data on the test duration was not available to the authors and it is therefore not possible to assess whether any of the tests presented in the following fall outside the 10 to 60 seconds test duration recommended by the ISRM (1985) and would require that they be invalidated.

The size of the sample for the tests performed on H2, H3 and H4 is 174, 156 and 120, respectively.

# 2. ANALYSIS

The variation of  $I_{s(50)}$  with the D/D<sub>e</sub> ratio for the full data set of H2, H3 and H4 indurated materials is presented in Figure 5 to Figure 7. A marked scatter can be observed in Figure 5 to Figure 7; a similar observation for weak carbonate rocks was made by Abbs (1985). No clear trend is observed between the D/D<sub>e</sub> ratio and  $I_{s(50)}$  for tested geomaterials as is to be expected given the definition and aim of the point load strength index.

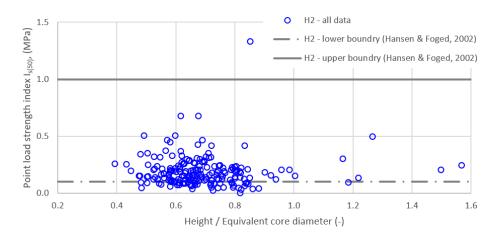


Figure 5. Point Load Strength Index  $I_{s(50)}$  vs D/D<sub>e</sub> for H2 indurated limestone and chalk.

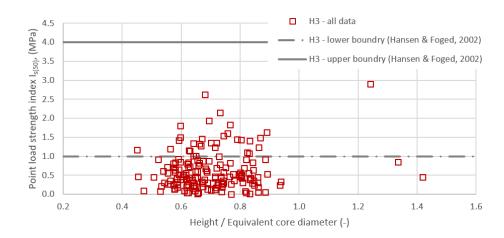


Figure 6. Point Load Strength Index  $I_{s(50)}$  vs D/D<sub>e</sub> for H3 indurated limestone and chalk.

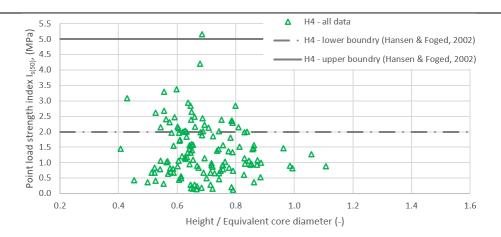


Figure 7. Point Load Strength Index  $I_{s(50)}$  vs D/D<sub>e</sub> for H4 indurated limestone.

# 2.1. Statistical distribution of the data

The statistical analysis of the point load strength index  $I_{s(50)}$  distribution is initially performed by visualising the data set on histograms. Normal and lognormal distributions are considered, as these are the most commonly used data distributions for this type of analysis. A qualitative assessment of the frequency diagrams on the left hand side of Figure 8 may suggest that the lognormal distribution has the potential to represent the data set well; however, the variation of the  $I_{s(50)}$  within each degree of induration does not seem to be symmetrical around the mean, neither for normal nor lognormal distribution: this would suggest an inherent uncertainty in terms of determining the variability of the  $I_{s(50)}$  within a specific degree of induration and specimen size.

The inverse standard normal cumulative distribution function for different indurations of limestone and chalk is shown in Figure 9 to aid the determination of the data distribution. The  $I_{s(50)}$  data for H2 indurated materials nearly plots on a straight line for both the normal and the lognormal distribution. Data sets for H3 and H4 indurations do not plot on a straight line for the normal distribution and seem to be better represented by a lognormal distribution.

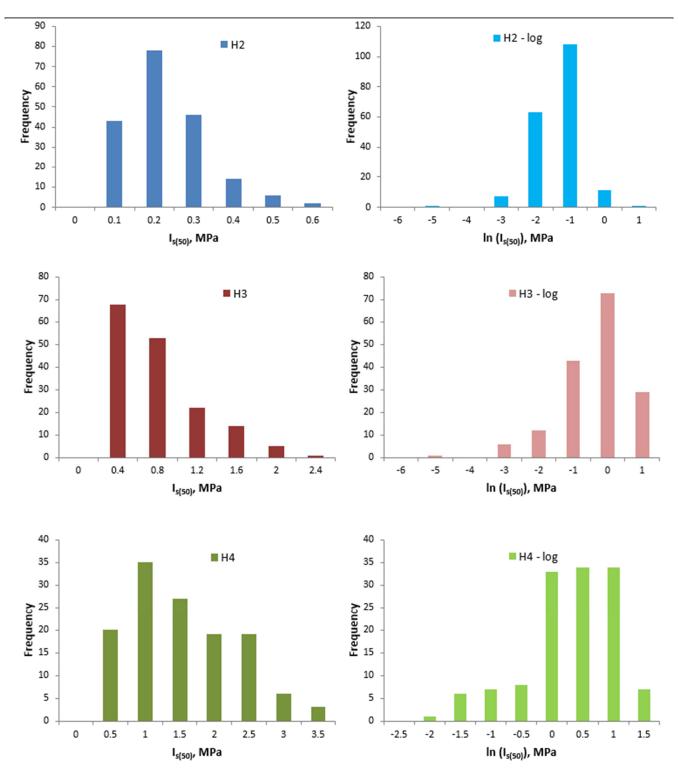


Figure 8. Histograms for different indurations of limestone and chalk to verify the distribution of the test data assuming normal (left) and lognormal (right) distributions.

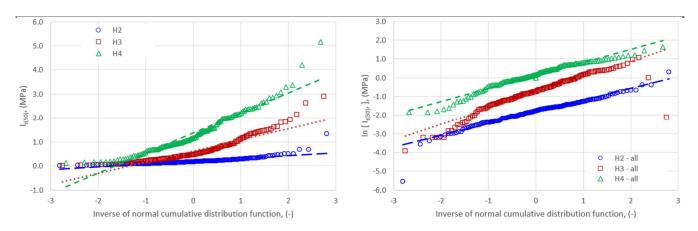


Figure 9. Inverse standard normal cumulative distribution function for different indurations of limestone and chalk to verify the distribution of the test data assuming normal (left) and lognormal (right) distributions.

## 2.2. Dependence of coefficient of variation on D/De ratio

The analysed tests were performed on specimens retrieved from different depths, however for the purpose of this analysis it is assumed that the weak rock strength represented by point load strength index  $I_{s(50)}$  varies with the degree of induration and is independent from the sampling depth.

Usually, it is assumed that coefficient of variation (CoV) provides a measure of dispersion of the data which is independent of the mean value. The CoV is calculated for normal and lognormal distribution of the of the  $I_{s(50)}$  data set excluding outliers defined test results values having more than ±3 standard deviations (three-sigma rule) from the mean value (Uzielli et al., 2007). In Figure 10 the CoV is plotted against the discrete mean values of D/D<sub>e</sub> for different ranges of the degree of induration of limestone and chalk. The most numerous data set consists of specimens with D/D<sub>e</sub> ratio in the range 0.6-0.7 for each degree of induration and is therefore expected to be less affected by the statistical estimation error.

The CoV calculated for the lognormal is generally higher than that of the normal distribution. Based on Figure 10 one could infer that for material with H3 and H4 induration the variability of  $I_{s(50)}$  decreases vs D/D<sub>e</sub> when this ratio is above 0.8, while this does not appear to be the case for H2 indurated material. Petrella (2001) found that the CoV reduces significantly within the D/D<sub>e</sub> ratio range of 0.6 to 0.8 for various types of isotropic and anisotropic rocks, such as gneiss, granite and marble and that the curves have a downward concavity, which is only partly consistent with the data set assessed herein.

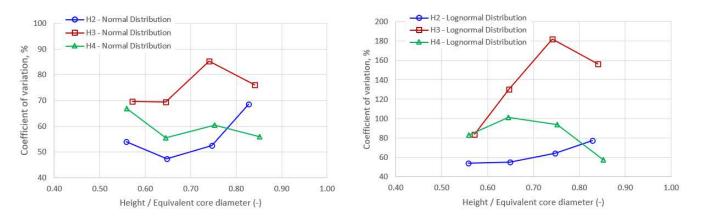


Figure 10. CoV vs D/De based on normal (left) and lognormal (right) distributions.

# 3. CONCLUSIONS

The results of circa five hundred point load tests carried out on specimens of Danian limestones and Maastrichtian chalk collected from two sites on the Danish island of Zealand were presented together with moisture content and bulk density. Notable limitations of the data set are the unavailability of tests for specimens with a  $D/D_e$  ratio >0.8 and missing information on the test duration.

It was found that the point load strength index values  $I_{s(50)}$  obtained for H2, H3 and H4 indurated materials do not strictly follow the indicative  $I_{s(50)}$  range values provided by Hansen & Foged (2002). Specifically, the results of 77% tests on strongly indurated (H4) specimens, 81% tests on indurated (H3) specimens and 20% on slightly indurated (H2) specimens fell outside the Danish literature ranges for Danian limestones.

The available data set does not allow to clearly attribute the scatter in the  $I_{s(50)}$  values to the specimen size expressed by  $D/D_e$  ratio, although it would be expected to see a lower variability  $I_{s(50)}$  when the specimen size is larger for the analysed degrees of induration of the tested limestone and chalk specimens. It shall be noted that the indicative ranges of compressive strength  $\sigma_c$  for the different degrees of induration are quite wide as the maximum value (high degree of induration) is 4 to 5 times larger than the lowest one (low degree of induration) as shown in Table 1. The scatter of the point load strength index  $I_{s(50)}$  with the induration decreases with increasing induration. Therefore it is presumed that the variability in the  $I_{s(50)}$  within the analysed test results is affected more by the aleatory uncertainty (Uzielli et al., 2007) of the data due to inherent heterogeneity of the limestone and chalk strength, than by the test specimen size.

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